

Separable Equations

Mathematics

Separating the variables, and the solutions that step loses.

For $dy/dx = f(x)g(y)$, where the variables come apart.

1 Check that it separates

The right side must factor as a function of x times one of y .

2 Separate the variables

Write $dy/g(y) = f(x) dx$, all the y one side and all the x the other.

3 Integrate both sides

One constant of integration is enough. Put it on the x side.

4 Solve for y if you can

An implicit solution is still a solution. Do not force an explicit form.

5 Recover the lost solutions

Dividing by $g(y)$ discarded the constant solutions where $g(y) = 0$.

Worked: $dy/dx = xy$ gives $\ln|y| = x^2/2 + C$, so $y = A e^{(x^2/2)}$, with $A = 0$ the solution that step two threw away.