First Isomorphism Theorem

Mathematics

The result that identifies every homomorphic image as a quotient.

For a homomorphism φ from G to H, the quotient of G by the kernel is isomorphic to the image of φ.

G / ker φ [illegible] im φ

Reduction [illegible] → $[illegible]_n$ — **[illegible]/n[illegible] [illegible] $[illegible]_n$**

The kernel is n[illegible].

Determinant on GL(n) — **GL/SL is the scalars**

Kernel is SL(n).

φ injective — **G is isomorphic to im φ**

Trivial kernel.

THE COMMON MISTAKE

× Writing G / im φ is isomorphic to ker φ

✓ G / ker φ is isomorphic to im φ

The kernel sits in the domain and the image in the codomain. Swapping them gives a statement whose two sides need not even be groups.

This is why kernels matter: every quotient of G is G by some kernel, and every homomorphic image of G is a quotient of G.