

Expectation & Variance

Mathematics

The algebra of means and variances, and where independence is needed.

a and b are constants; X and Y are random variables.

Expression	Equals
$E[aX + b]$	$a E[X] + b$
$E[X + Y]$	$E[X] + E[Y]$, always
$E[XY]$, X and Y independent	$E[X] E[Y]$
$\text{Var}(X)$	$E[X^2] - (E[X])^2$
$\text{Var}(aX + b)$	$a^2 \text{Var}(X)$
$\text{Var}(X + Y)$, independent	$\text{Var}(X) + \text{Var}(Y)$
$\text{Var}(X + Y)$, in general	$\text{Var } X + \text{Var } Y + 2\text{Cov}(X, Y)$

Expectation is linear with no independence assumption at all. Variance is not, and that is precisely where independence is needed.