Exact Equations

The cross-partial test, and what to do when it fails.

Mathematics

M dx + N dy = 0 is exact when the partial of M in y equals the partial of N in x, and then it is dF = 0.

$$\partial M/\partial y = \partial N/\partial x \Rightarrow F(x, y) = C$$

$2xy\,dx + x^2\,dy$ — **Exact, $F = x^2y$**

Both partials are 2x.

$y\,dx - x\,dy$ — **Not exact**

1 is not −1.

Test fails — **Seek μ(x) or μ(y)**

An integrating factor.

THE COMMON MISTAKE

× **Testing whether ∂M/∂x equals ∂N/∂y**

✓ **Testing whether ∂M/∂y equals ∂N/∂x**

It is the cross partials that must agree, since both equal the mixed second partial of F. The same-variable partials say nothing at all.

Find F by integrating M with respect to x, then differentiating in y and matching N. The constant of integration is a function of y.