Euler & Hamiltonian Paths

Mathematics

Two similar-sounding conditions with wildly different difficulty.

Euler path	Hamiltonian path
Uses every edge exactly once	Visits every vertex exactly once
Exists iff 0 or 2 vertices have odd degree	No simple criterion is known
A circuit needs every degree even	Deciding it is NP-complete
The graph must be connected	Dirac: every degree ≥ n/2 suffices
Decidable in linear time	Ore's condition is also sufficient
Fleury's or Hierholzer's algorithm finds one	Both are sufficient, never necessary

THE DIFFERENCE THAT MATTERS

Edges are easy and vertices are hard. Euler has a clean degree test; Hamilton has nothing comparable.

Dirac's theorem needs n ≥ 3 and every degree at least n/2. It guarantees a Hamiltonian cycle exists but does not construct one.