Epsilon-Delta Limits

Mathematics

The definition of a limit, with the order of the quantifiers made explicit.

lim f(x) = L at a means: for every ε > 0 there is a δ > 0 with |f(x) − L| < ε whenever 0 < |x − a| < δ.

$$\forall \varepsilon \; \exists \delta : 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

f(x) = 3x, a = 2, L = 6 — **Take δ = ε/3**

Linear, so δ scales.

The order of ε and δ — **ε first, δ second**

δ may depend on ε.

Why 0 < |x − a| — **f(a) is irrelevant**

The point is excluded.

THE COMMON MISTAKE

× Choosing δ first and then finding an ε

✓ ε is given; δ is what you construct

The order of the challenge is the entire content of the definition. Reversed, it becomes a statement true of almost every function.

δ may depend on both ε and a. When a single δ works for every a at once, the function is uniformly continuous.