Eigenvalues & Eigenvectors

Mathematics

The full procedure, plus the checks that catch an arithmetic slip.

Av = λv for some nonzero v: directions the map only stretches.

1. **Characteristic polynomial**
   Compute det(A − λI) and expand it as a polynomial in λ.

2. **Solve det(A − λI) = 0**
   The roots are the eigenvalues, each with an algebraic multiplicity.

3. **Find each eigenvector**
   Row reduce (A − λI)v = 0 and read off a basis of the null space.

4. **Compare multiplicities**
   Geometric multiplicity is that nullity, and never exceeds algebraic.

5. **Check your arithmetic**
   The eigenvalues sum to the trace and multiply to the determinant.

An n×n matrix has n eigenvalues over ℂ counted with multiplicity, but it may have far fewer independent eigenvectors.