

Dot & Cross Product

Mathematics

Two products: different outputs, different geometry, different laws.

Dot product

- Output is a scalar
- $u \cdot v = |u||v| \cos \theta$
- Zero exactly when perpendicular
- Commutative: $u \cdot v = v \cdot u$
- Defined in every $\mathbb{R}^n$
- Gives lengths, angles and projections

Cross product

- Output is a vector
- $|u \times v| = |u||v| \sin \theta$
- Zero exactly when parallel
- Anticommutative: $u \times v = -(v \times u)$
- Defined only in $\mathbb{R}^3$
- Gives areas, normals and moments

THE DIFFERENCE THAT MATTERS

The dot product measures how far two vectors agree; the cross product measures how far they do not.

For nonzero vectors, $u \cdot v = 0$ and $u \times v = 0$ cannot both hold: perpendicular and parallel are exclusive.