Differentiable vs Continuous

Mathematics

One implies the other, in exactly one direction.

Continuous at a

- lim f(x) = f(a) as x approaches a
- No jump, no hole, no blow-up
- Requires no tangent line
- |x| is continuous everywhere
- The weaker condition

Differentiable at a

- The difference quotient has a limit
- A unique tangent line exists
- Implies continuity at a
- |x| fails this at x = 0
- The stronger condition

THE DIFFERENCE THAT MATTERS

Differentiable implies continuous. The converse is false, and spectacularly so.

The Weierstrass function is continuous everywhere and differentiable nowhere. Continuity buys far less than intuition suggests.