

Cyclic Groups

Mathematics

The simplest groups there are, and the complete list of their subgroups.

A cyclic group is generated by a single element: every member is a power of that generator.

ORDER OF $\langle g \rangle$

The order of g itself

GENERATORS OF $\langle g \rangle_n$

$\varphi(n)$ of them

SUBGROUPS OF $\langle g \rangle_n$

One per divisor of n

- Written $\langle g \rangle$, where g is a generator
- Every cyclic group is abelian
- Infinite cyclic groups are isomorphic to $\mathbb{Z}$
- Finite ones of order n are isomorphic to $\mathbb{Z}_n$
- Every subgroup of a cyclic group is cyclic
- $\mathbb{Z}_n$ has exactly one subgroup per divisor of n
- $\mathbb{Z}_n$ has $\varphi(n)$ generators, φ being Euler's totient

LOOK FOR IT

$\mathbb{Z}_6$ has subgroups of order 1, 2, 3 and 6 — one for each divisor of 6.

A group of prime order is cyclic and has no proper nontrivial subgroup, by Lagrange. There is exactly one such group per prime.