

Cosets & Normal Subgroups

Mathematics

What cosets always do, and the extra condition that makes them a group.

Cosets in general

- gH is the set of all gh with h in H
- They partition G into equal-sized blocks
- Left and right cosets may differ
- Not a subgroup unless g lies in H
- Their number is the index $[G : H]$

Normal subgroups

- $gH = Hg$ for every g in G
- Equivalently $gHg^{-1} = H$ for every g
- The cosets then form a group G/H
- Any subgroup of index 2 is normal
- Every kernel of a homomorphism is normal

THE DIFFERENCE THAT MATTERS

Cosets always partition the group. They form a group only when the subgroup is normal.

In an abelian group every subgroup is normal. The smallest failure elsewhere is S_3 , where the subgroup generated by one swap is not.