

Continuity Theorems

Mathematics

The four results that a closed bounded interval buys you.

Every one of these needs a closed and bounded interval.

Theorem	Hypotheses	Conclusion
Intermediate value	f continuous on $[a, b]$	Attains every value between
Extreme value	f continuous on $[a, b]$	Attains a maximum and minimum
Boundedness	f continuous on $[a, b]$	f is bounded on $[a, b]$
Heine-Cantor	f continuous on $[a, b]$	f is uniformly continuous
Brouwer in 1D	f maps $[a, b]$ into itself	Some c satisfies $f(c) = c$

Drop closedness and all four fail at once: $1/x$ on $(0, 1]$ is continuous, unbounded, attains no maximum and is not uniformly continuous.