Conditional Probability

Mathematics

Probability rescaled to the world in which one event has happened.

The probability of A given B rescales the joint probability to the world in which B has happened.

$$P(A \mid B) = P(A \cap B) / P(B), \text{ for } P(B) > 0$$

$P(A \cap B)=0.2$, $P(B)=0.5$ — $P(A \mid B) = 0.4$

$0.2 \div 0.5$.

A and B independent — $P(A \mid B) = P(A)$

B tells you nothing.

Chain rule — $P(A \cap B) = P(A|B)P(B)$

The same line, rearranged.

THE COMMON MISTAKE

× Reading $P(A \mid B)$ as though it were $P(B \mid A)$

✓ Keeping the conditioning event fixed

P(rain | clouds) and P(clouds | rain) are very different numbers. Reversing the conditioning is the base-rate fallacy.

Independence means $P(A \cap B) = P(A)P(B)$. Mutually exclusive events are not independent — knowing one occurred rules the other out.