

Compactness

Mathematics

The equivalent characterisations, and where the familiar one stops working.

Compactness is finiteness for infinite sets.

Statement	Status
Every open cover has a finite subcover	The definition itself
Closed and bounded	Equivalent in $\mathbb{R}^n$ only
Sequentially compact	Equivalent in metric spaces
Continuous image of a compact set	Compact, always
Continuous f on a compact set	Attains its max and min
Compact set in a metric space	Always closed and bounded

Closed and bounded means compact in $\mathbb{R}^n$ and nowhere else. The closed unit ball of an infinite-dimensional space is not compact.