

# The Chain Rule

Mathematics

Differentiating a composition, and the factor everyone drops.

**The derivative of a composition is the outer derivative at the inner function, times the inner derivative.**

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} \sin(3x^2)$$

Outer sin, inner  $3x^2$ .

$$6x \cos(3x^2)$$

$$\frac{d}{dx} e^{-x^2}$$

Inner derivative  $-2x$ .

$$-2x e^{-x^2}$$

$$\frac{d}{dx} (\ln x)^5$$

Power outer, ln inner.

$$5(\ln x)^4 / x$$

## THE COMMON MISTAKE

✗  $\frac{d}{dx} \sin(3x) = \cos(3x)$

✓  $\frac{d}{dx} \sin(3x) = 3 \cos(3x)$

Forgetting the inner derivative is the commonest calculus slip. Differentiate the outer function, then multiply by the derivative of the inside.

In Leibniz form  $dy/dx = (dy/du)(du/dx)$ . The  $u$  must be a genuine function of  $x$  for that cancellation to mean anything.