Central Limit Theorem

Mathematics

What becomes normal, under what conditions, and what does not.

For independent identically distributed variables with finite variance, the sample mean standardises to a normal.

$$(M_n - \mu)/(\sigma/\sqrt{n}) \rightarrow \text{Normal}(0, 1)$$

n = 30, any shape A rule of thumb.	**Mean roughly normal**
Standard error Shrinks like 1/√n.	**σ/√n**
Infinite variance Cauchy has no mean.	**Theorem does not apply**

THE COMMON MISTAKE

× **Claiming the data themselves become normal**

✓ **The sample mean becomes normal**

The population distribution does not change at all. It is the sampling distribution of the mean that approaches normality as n grows.

It needs independence, identical distribution and finite variance. The n ≥ 30 guideline is folklore, not part of the theorem.