Cauchy Sequences

Mathematics

Convergence without naming a limit, and what completeness buys you.

Convergent

- Terms approach one specific limit L
- The definition names L explicitly
- You must know the limit to apply it
- Every convergent sequence is Cauchy
- Meaningful in any metric space

Cauchy

- Terms eventually approach each other
- The definition names no limit at all
- Testable without knowing the limit
- Cauchy implies convergent only if complete
- Every Cauchy sequence is bounded

THE DIFFERENCE THAT MATTERS

In □ the two conditions are equivalent, and that equivalence is exactly what completeness means.

In □ the decimal truncations of √2 form a Cauchy sequence with no rational limit. □ is not complete.