Bolzano-Weierstrass

Mathematics

Every bounded sequence has a convergent subsequence, and what that does not say.

Every bounded sequence of real numbers has at least one convergent subsequence.

Bounded in $\mathbb{R}^n$ ⇒ some subsequence converges

$a_n = (-1)^n$ — **Even terms → 1**

A subsequence works.

$a_n = \sin n$ — **Subsequence converges**

Bounded by 1.

$a_n = n$ — **No such subsequence**

Unbounded sequence.

THE COMMON MISTAKE

× **Concluding the sequence itself converges**

✓ **Concluding only that some subsequence does**

$(-1)^n$ is bounded and divergent. The theorem promises a convergent subsequence and it promises nothing more than that.

In $\mathbb{R}^n$ this is equivalent to compactness of closed bounded sets. In infinite-dimensional spaces it fails outright.