The Binomial Theorem

Mathematics

The expansion of a binomial power, and where Pascal's triangle comes from.

The expansion of $(x + y)^n$ sums $C(n, k)\, x^{n-k}\, y^k$ over k from 0 to n.

$$(x + y)^n = \sum C(n, k)\, x^{n-k}\, y^k$$

$(x + y)^3$ → $x^3 + 3x^2y + 3xy^2 + y^3$

Row 3 of Pascal.

x^2y^3 in $(x + y)^5$ → $C(5, 2) = 10$

Choose where the x go.

$\sum C(n, k)$ over all k → 2^n

Set x = y = 1.

THE COMMON MISTAKE

× $(x + y)^n = x^n + y^n$

✓ **Every cross term appears as well**

Only n = 1 makes that true. Squaring gives $x^2 + 2xy + y^2$; the middle term counts the two ways of picking one x and one y.

Pascal's triangle: $C(n, k) = C(n - 1, k - 1) + C(n - 1, k)$. Each entry is the sum of the two directly above it.