

Absolute vs Conditional

Mathematics

Two kinds of convergent series, one of which survives rearrangement.

Absolute convergence

- $\sum |a_n|$ itself converges
- It implies $\sum a_n$ converges
- Rearranging changes nothing
- Products behave as you would hope
- $\sum (-1)^n/n^2$ is absolutely convergent

Conditional convergence

- $\sum a_n$ converges but $\sum |a_n|$ does not
- Fragile: the order of terms matters
- Rearrangeable to any sum at all
- Only the alternating test applies
- $\sum (-1)^n/n$ is conditionally convergent

THE DIFFERENCE THAT MATTERS

Absolute convergence is a property of the terms; conditional convergence is a property of their order.

Riemann's rearrangement theorem: a conditionally convergent series can be rearranged to reach any real number, or to diverge.